7. Appendix 1: Terms of reference

Terms of Reference

Information for Climate Risk Management
(KAP II Component 1.4.0)
Procurement Plan ref. FS5
Kiribati Adaptation Program – Pilot Implementation Phase (KAP II)

1. General background

Kiribati is one of the most vulnerable countries in the world to the effects of climate change and sea level rise. Most of the land in urban Tarawa is less than 3 meters above sea level; the island has an average width of only 450 meters, rendering retreat adaptation options untenable. This situation is typical of most islands in the country. The islands are exposed to periodic storm surges and droughts, particularly during La Niña years, although they lie outside the cyclone path. Already, Kiribati is becoming increasingly vulnerable to climate events due to its high population concentration, accelerated coastal development, shoreline erosion, and rising environmental degradation. By 2050, if no adaptation measures are undertaken, Kiribati could face economic damages due to climate change and sea level rise of US$8-$16 million a year, equivalent to 17-34 percent of its 1998 GDP.

To address these rising risks, the Government of Kiribati is undertaking an Adaptation Program, supported by the World Bank, the Global Environmental Facility, AusAID and NZAID, the Japan PHRD Climate Change Fund, UNDP, and a parallel project by the EU. The key goal is to reduce Kiribati's vulnerability to climate change, climate variability and sea level rise. This Kiribati Adaptation Program (KAP) is planned to be implemented in three phases:

- **Phase I: Preparation** (2003-2005, completed). This phase began the process of mainstreaming adaptation into national economic planning and identified priority pilot investments for Phase II. It also involved an extensive process of national consultation. The project was closely linked with the preparation of the 2004-07 National Development Strategy and Ministry Operational Plans.

- **Phase II: Pilot Implementation** (2006-2009). This current phase is the focus of this TOR. Its objective is to implement pilot adaptation measures, and consolidate the mainstreaming of adaptation into national economic planning.

- **Phase III: Expansion** (2009-2015). This phase would gradually scale up the investments piloted under Phase II to cover all major islands and vulnerable sectors of Kiribati.

The key objective of the current Pilot Implementation Phase, KAP II, is to develop and demonstrate the systematic diagnosis of climate-related problems and the design of cost-effective adaptation measures, while continuing the integration of climate risk awareness and responsiveness into economic and operational planning. Lessons learned from KAP-II would be used to plan the long-term national response to climate change envisaged for 2009/10 onwards. KAP II includes the following components:

Component 1: Policy, planning, and information. This component supports three core elements of all adaptation efforts in Kiribati. The first element is awareness raising and consultation. The second element is policy coordination and planning, including support to the new National Strategic Risk Management Unit in the Office of Te Beretitenti; continued mainstreaming into Ministry Operational Plans; and integration of adaptation into population and resettlement programs. The third element is to generate scientific climate risk information and refurbish the capacity of the Meteorological Office.

Component 2: Land use, physical structures, and ecosystems. This component will contribute to reducing the vulnerability of the coastline including key public assets and ecosystems, shifting the coastal management practice from a reactive, single technique approach to repairing damage as it occurs, to a preventative and more technically varied risk mitigation strategy, including more attention for environmental sustainability. More specifically, the component would support the development and application of improved risk diagnosis and response methods, and improvements in planning and permitting processes to guide coastal zone activities, including regulatory adjustments, awareness raising and enforcement, and economic and environmental monitoring. Secondly, the component will produce design and construction guidelines, and apply them to a sample of public assets that are at risk, including the national hospital and vulnerable coastal areas. Thirdly, the component includes monitoring and pilot activities to protect and restore coastal ecosystems and biodiversity affected by climate change, climate variability and sea level rise, including the detrimental effects of current adaptation practices.

Component 3: Freshwater resources. This component supports the development and management of freshwater resources to reduce their vulnerability to climate variability and climate change. It will provide assistance to update the national water policy, improve water resource management, and revise building codes to enhance opportunities for rainwater catchment and storage. Given that water management problems are most acute on the central island, Tarawa, the component will also support the preparation of a master plan for water resources on Tarawa, pilot projects to identify and increase water resources in freshwater lenses, and a public awareness and education campaign to change user attitudes. On the outer islands, the component focuses on water resource assessments and improvements in the water supply system in selected locations, including attention for non-polluting sanitation systems.

Component 4: Capacity at island and community level. This component provides assistance to the Ministry of Internal and Social Affairs (MISA) to include adaptation in the Outer Island Profiles, and training on climate risk management for local governments. Furthermore, it supports a program of small-scale adaptation investments in two selected outer islands, identified through participatory planning and implemented directly by communities.

Component 5: Program management. This project component provides overall support to the project, including program management, accounting, procurement, and running costs of the Program Management Unit. It will also support the evaluation of KAP-II in view of the design of the next phase of GoK adaptation efforts.

2. Specific background for this assignment

Adaptation to the impacts of climate change in atoll countries requires evaluation of a range of physical and planning measures and subsequent implementation of optimal solutions, by the government and other stakeholders.

For coastal management, Kiribati currently relies on a narrow range of engineering solutions (seawalls) to coastal hazards that are under-engineered with regard to future climate change and construction techniques. In the water sector, climate risks are not included in problem diagnosis, options analysis, and design of system upgrades. Building codes do not reflect climate standards.

Constraints to improving the range and technical robustness of engineered solutions include: limited technical design expertise; limited awareness and testing of alternative options; current construction practices; but also the lack of technically defensible design parameters that take account of present and future climate change (e.g., storm surge, wave height, maximum water levels, and return period of extreme rainfall events and droughts).

3. Objective of the assignment

The objective of this assignment is to develop climate risk information to be adopted as national standard for options analysis and technical design work ("climate proofing parameters"), particularly regarding coastal and water related issues.

4. Tasks

It is expected that the TA will establish, through data analysis, fieldwork and modelling, a range of extreme water levels and drought and flood return periods that underpin adaptation designs. The analysis should certainly cover Tarawa (for which data are most readily available, but to the extent possible also other islands (for instance those with a long-term meteorological data record).

The analysis should include:

1. Analysis and estimates of storm surge that accord with the 1%, 2% and 10% annual exceedence probability (AEP). Such analysis should give explicit consideration of:
    a. The inverse barometric effect.
    b. Wind stress.
    c. Wave setup.
2. Analysis and estimates of maximum at-shoreline wave heights that correspond with the 1%, 2% and 10% AEP.
3. Analysis and estimates of maximum wave run-up levels for a range of differing foreshore slope angles.
4. Generation of storm surge, wave height and run-up estimates for a range of different island exposures that should at least include:
    a. Windward ocean shoreline.
    b. Leeward ocean shoreline.

   c. Windward lagoon shoreline.
   d. Leeward lagoon shoreline.
   e. Gross differences in reef platform width.

5. Analysis of intensity of extreme rainfall events with a return period of 10, 50 and 100 years, as well as droughts with a return period of 10, 50, and 100 years.
6. Identify and procure relevant datasets (e.g., wind, atmospheric pressure, sea level) to support analysis and collect new data (e.g., waves) where appropriate.
7. In collaboration with the GoK CCST, identify combined scenarios of sea level change, extreme water levels, the risk of extreme rainfall, as well as drought, for three time horizons 2025, 2050 and 2100.
8. All estimates of water level heights, extreme rainfall and droughts should be generated with reference to a known survey datum.
9. Conduct in-house trainings to GoK CCST on the tools and methods needed for analysis of data and generation of climate data trends.
10. Conduct a workshop to all stakeholders to present conclusions of his assignment.
11. Work with Govt agencies to develop a user-friendly database framework for data storage and updating.
12. Work with Awareness Technical Assistants to integrate climate risk information into awareness products.

5. Outputs

A document describing all the information listed above, to be discussed with all government agencies that would be using such information (and those who would be collecting the data to update it). Information must be presented in a very user-friendly way, focusing only on the essentials and including an explanation of how they could be applied in different contexts. Scientific details should only be inserted in technical annexes.

A draft report will be presented to all key agencies for questions and comments, so that the contents as well as the presentation can be fine-tuned to the needs of the end users of the information.

A Database framework to archive, manage and update data gathered in this assignment.

6. Arrangements

Responsible to: KAP II Project Director

Co-operating/Collaborating with the following Government of Kiribati agencies: National Strategic Risk Management Unit; Climate Change Study Team; Ministry of

Public Works and Utilities; Meteorological Service; as well as the Tidal Facility in Betio.

7. Timing

Three months work have been allocated for this activity. It is envisaged that the activity will require at least a 3-4 week visit to Kiribati to undertake field data collection where appropriate and to liaise with relevant agencies. At the end of this visit, a workshop will be organized with key users of the information to inform them of some preliminary results and have an exchange of ideas about the uses and packaging of the information. This should include the international technical assistant working on the development of a risk diagnosis and response process for coastal management (component 1.3.2) and if possible the technical assistant working on risk analysis and design capacity for coastal hazard protection of key public assets (2.2.1).

The activity is scheduled for the fourth quarter of 2007 and first quarter 2008.

Annex B: Consultant's Personnel

Name	Position
1. Mr. Douglas Ramsay	Leading Consultant (NIWA)
2. Dr. Rob Bell	Consultant (NIWA)
3. Dr. Richard Gorman	Consultant (NIWA)
4. Mr. Craig Thompson	Consultant (NIWA)
5. Dr. Scott Stephens	Consultant (NIWA)

Annex C: Consultant's Reporting Obligations

The Consultant shall submit to the Client the reports in the form and within the time periods specified in Annex C, "Consultant's Reporting Obligations"

1. **Provide technical analysis and report on climate risk information as detailed.** ***Timing Month 2***
   a. Analysis and estimates of storm surge that accord with the 1%, 2% and 10% annual exceedence probability (AEP). Such analysis should give explicit consideration of:
      a. The inverse barometric effect.
      b. Wind stress.
      c. Wave setup.
   b. Analysis and estimates of maximum at-shoreline wave heights that correspond with the 1%, 2% and 10% AEP.
   c. Analysis and estimates of maximum wave run-up levels for a range of differing foreshore slope angles.
   d. Generation of storm surge, wave height and run-up estimates for a range of different island exposures that should at least include:
      a. Windward ocean shoreline.
      b. Leeward ocean shoreline.

c. Windward lagoon shoreline.

d. Leeward lagoon shoreline.

e. Gross differences in reef platform width.

e. Analysis of intensity of extreme rainfall events with a return period of 10, 50 and 100 years, as well as droughts with a return period of 10, 50, and 100 years.

f. In collaboration with the GoK CCST, identify combined scenarios of sea level change, extreme water levels, the risk of extreme rainfall, as well as drought, for three time horizons 2025, 2050 and 2100.

g. All estimates of water level heights, extreme rainfall and droughts should be generated with reference to a known survey datum.

2. Identify and procure datasets and develop user-friendly database framework for data storage and updating

Timing Month 2

h. Identify and procure relevant datasets (e.g., wind, atmospheric pressure, sea level) to support analysis and collect new data (e.g., waves) where appropriate.

i. Work with Government agencies to develop a user-friendly database framework for data storage and updating.

3. Conduct training and workshop

Timing Month 3

j. Conduct in-house trainings to GoK CCST on the tools and methods needed for analysis of data and generation of climate data trends.

k. Conduct a workshop to all stakeholders to present conclusions of his assignment.

8. Appendix 2: Glossary of Terms

Adaptation to climate change	Undertaking actions to minimise threats or to maximise opportunities resulting from climate change and its effects.
Anthropogenic	Produced by human beings or resulting from human activities.
Anthropogenic emissions	Emissions of *greenhouse gases*, greenhouse gas *precursors* and *aerosols* associated with human activities. These include burning of fossil fuels for energy, deforestation, and land-use changes that result in a net increase in emissions.
AOGCM	Acronym for *atmosphere-ocean general circulation model*.
AR4	Acronym for IPCC *Fourth Assessment Report* 2007
ARI	Acronym for Average Recurrence Interval. Same as *return period*.
Atmosphere-ocean general circulation model (AOGCM)	A comprehensive *climate model* containing equations representing the behaviour of the atmosphere, ocean and sea ice and their interactions.
Carbon dioxide (CO_2)	A naturally occurring gas and the principal anthropogenic greenhouse gas.
CFC	Acronym for *Chlorofluorocarbon.*
Climate	The "average weather", over a period of time ranging from months to thousands or millions of years. The classical period for calculating a "climate normal" is 30 years.
Climate change	A statistically significant variation in either the mean state of the climate or in its variability, persisting for an extended period (typically decades or longer).
Climate model	A numerical representation (typically a set of equations programmed into a computer) of the *climate system*. The most complex and complete climate models are known as *General Circulation Models* (below).
Climate prediction	An attempt to provide a most likely description or estimate of the actual future evolution of the *climate*.
Climate projection	A potential future evolution of the climate in response to an emission or concentration *scenario* of *greenhouse gases* and *aerosols*. Often based on a simulation by a *climate model*.
Climate variability	Variations of the *climate* (e.g., of the mean state, standard deviations and extremes) on all temporal and spatial scales beyond those of individual weather events.
Drought	Drought is a relative term. It is generally associated with abnormally long dry periods which lead to a lack of rain sufficient to meet normal requirements. There are several definitions of drought which are based on meteorological, hydrological, agricultural and economic considerations.
Drought occurrence parameter (θ)	The average number of droughts per year derived from the historical record.
Drought severity index	An index of drought based on precipitation anomalies over a prescribed time period (e.g., months). The index allows for preceding deficits and surpluses of precipitation.

El Niño	A significant increase in sea surface temperature over the eastern and central equatorial Pacific that occurs at irregular intervals, generally ranging between two and seven years. Associated changes occur in atmospheric pressure patterns and wind systems across the Pacific. These can lead to changes in seasonal rainfall and temperature in parts of Australia and New Zealand.
El Niño Southern Oscillation (ENSO)	Term coined in the early 1980s in recognition of the intimate linkage between *El Niño* events and the *Southern Oscillation*, which, prior to the late 1960s, had been viewed as two unrelated phenomena. The interactive global ocean-atmosphere cycle comprising El Niño and La Niña is often called the ENSO cycle.
Extreme weather event	An event that is rare at a particular place. Rare would normally be defined as rare as or rarer than the 10th or 90th percentile.
ENSO	Acronym for *El Niño Southern Oscillation*.
General Circulation Model (GCM)	A global, three-dimensional computer model of the *climate system*, which can be used to simulate the general circulation and climate of the atmosphere and ocean, and particularly human-induced climate change. GCMs are highly complex and they represent the effects of such factors as reflective and absorptive properties of atmospheric water vapour, greenhouse gas concentrations, clouds, annual and daily solar heating, ocean temperatures and ice boundaries. GCMs include global representations of the atmosphere, oceans, and land surface.
GCM	Acronym for *General Circulation Model* or *Global Climate Model.*
Global Climate Model (GCM)	The same as *General Circulation Model.*
Global surface temperature	The global surface temperature is the area-weighted global average of: (i) the sea-surface temperature over the oceans (i.e., the subsurface bulk temperature in the first few metres of the ocean); and (ii) the surface-air temperature over land at 1.5 m above the ground.
Global warming	Usually used to refer to the rise of the earth's surface temperature predicted to occur as a result of increased emissions of *greenhouse gases*.
Greenhouse effect	An increase in the temperature of the earth's surface and the lowest 8 km or so of the atmosphere, caused by the trapping of heat by *greenhouse gases*. Naturally occurring greenhouse gases cause a greenhouse effect at the earth's surface of about 30°C. Further temperature increases caused by *anthropogenic emissions* are termed the enhanced greenhouse effect.
Greenhouse gases	Gases in the earth's atmosphere that absorb and re-emit infra-red (heat) radiation. Many greenhouse gases occur naturally in the atmosphere, but concentrations of some (such as *carbon dioxide*, methane, and nitrous oxide) have increased above natural levels because of *anthropogenic emissions*.
Interdecadal Pacific Oscillation (IPO)	A long timescale oscillation in the Pacific Ocean-atmosphere system which shifts climate every one to three decades. The IPO has positive (warm) and negative (cool) phases. Positive phases tend to be associated with an increase in *El Niños*, and negative phases with an increase in *La Niñas*.

Intergovernmental Panel on Climate Change (IPCC)	The body established in 1988 by the World Meteorological Organisation (WMO) and the United Nations Environment Programme (UNEP) to objectively assess scientific, technical and socioeconomic information relevant to understanding the scientific basis of risk of human-induced climate change, its potential impacts and options for adaptation and mitigation.
IPCC	Acronym for Intergovernmental Panel on Climate Change.
IPO	Acronym for *Interdecadal Pacific Oscillation.*
La Niña	A significant decrease in sea surface temperature in the central and eastern equatorial Pacific that occurs at irregular intervals, generally ranging between two and seven years. La Niña is the cool counterpart to the *El Niño* warm event, and its spatial and temporal evolution in the equatorial Pacific is, to a considerable extent, the mirror image of El Niño. Like El Niño, there are associated changes in atmospheric pressures and wind systems across the Pacific, and related changes can occur in temperature and rainfall in parts of Australia and New Zealand.
Natural variability	Non-anthropogenic climate variability that may be irregular or quasi-cyclic. *El Niño-Southern Oscillation* is probably the best-known example of a natural oscillation of the climate system, but there are many others. Changes caused by volcanic eruptions and solar variations can also be considered “natural”.
Return period	The probable time period between repetition of *extreme weather events*, such as heavy rainfall or flooding, in a stationary climate (that is, a climate without global warming or other trends). In the case of rainfall, a return period is always related to a specific duration (e.g., 50-year return period of 24-hour extreme rainfall).
Scenario	A plausible and often simplified description of how the future may develop, based on a coherent and internally consistent set of assumptions about key driving forces.
SOI	Acronym for *Southern Oscillation Index*.
Southern oscillation	A multi-year low-latitude seesaw in sea level pressure, with one pole in the eastern Pacific and the other in the western Pacific/Indian Ocean region. This pressure seesaw is associated with a global pattern of atmospheric *anomalies* in circulation, temperature, and precipitation. Its opposite extremes are the *El Niño* and *La Niña* events.
Southern oscillation index	An index calculated from *anomalies* in the pressure difference between Tahiti and Darwin. Low negative values of this index correspond to *El Niño* conditions, and high positive SOI values coincide with *La Niña* episodes.
SRES scenarios	A set of *greenhouse gas* and *aerosol* emissions *scenarios* developed in 2000 by Working Group III of the *IPCC* and used, among others, as a basis for the climate projections in the IPCC’s 2001 Third Assessment Report.
TAR	Acronym for IPCC *Third Assessment Report* 2001

9. Appendix 3: Outline of the theory of extreme values

If a sample of N observations is selected at random from a population with a common probability distribution $P(x)$, then $P_N(X)$ is the probability that X is the largest value in the sample. Given that the largest values of n random samples of size N, then X_1, X_2, …, X_n also has a common probability distribution $P_N(X)$. The distribution of the largest values depends on the sample size N, and the form of the distribution $P(x)$.

The theory of extreme values has established that when N is very large, the probability distribution of the largest values is not very dependent on the exact form of the initial distribution, and can be approximated by a number of possible forms of extreme value distribution functions. In the analysis presented in this report, the extreme value distribution fitted to the sample data is the Generalised Extreme Value (GEV) distribution, and more especially on of the three forms of the GEV, the EV1 or Gumbel distribution. GEV distributions are commonly used in the analysis of environmental data.

The large sample EV1 distribution is given by:

$$\Pr ob(X \le x) = F(x) = \exp(-\exp(-(x-U)/a))$$

where U and a are the mode (or location) and dispersion (or scale) of the distribution estimated from the sample of extreme values, and $F(x)$ is the cumulative distribution function of the EV1 distribution. This relation provides the probability that the specified value for x is never exceeded. Values for U and a are provided in Table 3.1 for each location and rainfall duration.

Several methods are readily available to estimate these parameters: in this report L-moments are used. L-moments are modifications to the probability weighted moment procedure. Both methods provide identical results. In terms of L-moments the parameters of the EV1 distribution are:

$$a = \lambda_2 / \log(2)$$
$$U = \lambda_1 - a\gamma$$

where λ_1 and λ_2 are the first and second L-moment and γ is Eulers constant. L-moments are calculated from ordered samples of data, $x_1 \le x_2 \le .... \le x_n$. The first L-moment is simply the mean of the data sample, and the second L-moment is formed from the absolute mean difference statistic.

Table 3.1: Values of *U* and *a* for the different locations and rainfall durations.

Location		Duration 10m	20m	30m	1h	2h	6h	12h	24h	48h	72h
Banaba	**U**	17.99	28.34	33.73	54.36	63.62	83.42	102.31	114.65	137.25	157.06
	a	2.85	4.94	7.40	15.64	28.50	39.94	49.01	50.63	52.80	59.11
Tarawa	**U**	17.32	26.38	33.66	45.17	57.81	89.32	102.60	113.02	132.26	144.87
	a	4.96	8.09	8.53	12.09	15.93	29.47	36.35	41.59	47.20	50.57

		Duration 1-day	2-day	3-day			Duration 1-day	2-day	3-day
Fanning Island	**U**	97.6	126.9	143.6	**Christmas Island**	**U**	59.8	73.8	83.4
	a	33.0	38.0	45.1		**a**	47.1	56.2	67.3
Banaba	**U**	99.7	128.2	143.6	**Butaritari**	**U**	116.4	142.3	162.7
	a	39.1	44.9	52.3		**a**	44.9	57.0	64.1
Tarawa	**U**	95.5	120.2	134.3	**Beru**	**U**	77.9	95.9	108.7
	a	37.7	46.8	50.9		**a**	38.1	43.6	53.3
Arorae	**U**	97.7	119.7	135.9	**Kanton**	**U**	67.2	83.3	94.7
	a	30.9	39.2	46.6		**a**	39.8	49.0	57.9

In applying the theory, it is more usual to express probability in terms of the annual exceedances, given as $\Pr ob(X \geq x) = 1 - F(x)$, or in terms of the average recurrence interval, ARI, or return period *T*(*x*) such that, $T(x) = 1/(1 - F(x))$, and provides the average time interval for a rainfall of magnitude x to be at least equalled. Table 3.2 gives examples of the relationship between annual exceedance probability (AEP) and ARI. For example a 1 percent AEP, or a 1 in 100 AEP corresponds to an ARI of 100 years.

Table 3.2: Annual Exceedance Probabilities and Annual Recurrence Intervals.

AEP (%)	ARI (years)
1	100
2	50
5	20
10	10
20	5
50	2

For a given recurrence interval *T* the amount of rainfall or quantile estimate is given by:

$$X_T = U - a(\log_e(-\log_e(1-1/T))$$

10. Appendix 4: Phillips and McGregor index of drought severity

This drought severity index, proposed by Phillips and McGregor (1998), is based on the precipitation deficit-index method of Bryant et al. (1992), and uses monthly precipitation anomalies. The index is sufficiently flexible, having been tested by Phillips and McGregor, (1998) over a range of sub-monthly and monthly initiation and termination criteria. In this paper, a drought severity index, DSI_3 is derived for all Kiribati sites having at least 20 years of monthly rainfall observations. DSI_3 uses three-month drought onset and termination rule. The index allows for preceding precipitation deficits or surpluses in the formulation.

10.1 Drought onset

If the precipitation anomaly in month t is $X_t < 0$, and the anomalies in the preceding three-month period (i.e., X_{t-1}, X_{t-2}, X_{t-3}) are less than the three-monthly median (i.e., $\sum_{i=1}^{3} X_{t-i} < 0$), then a drought sequence begins. The precipitation anomaly, $|X_t|$, is the value of DSI_3 for month t. Clearly, a drought will not start if large precipitation surpluses exist over the preceding three-month period. In this formulation, precipitation anomalies are computed from monthly medians, rather than from monthly averages as in Phillips and McGregor (1998). Unlike the mean, the median is a robust statistic having a small influence from outliers and positively skewed precipitation distributions.

10.2 Drought continuation

Now if the precipitation anomaly in months t and t+1 are $-Y$ mm and $-Z$ mm respectively, then DSI_3 for month t+1 will be $Y+Z$ if and only if the total monthly precipitation over the preceding three-months is still less than the corresponding three-monthly median total. Should the rainfall in month t+1 be greater than the median (i.e., the anomaly is positive), then drought can only continue if $\sum_{i=1}^{3} X_{t-i} < 0$, in which case DSI_3 becomes $Y - Z$.

10.3 Termination

A drought sequence ends when the accumulated three-monthly median precipitation has been exceeded, i.e., $\sum_{i=1}^{3} X_{t-i} \geq 0$. When this occurs DSI_3 is reset to zero.

10.4 Standardisation

The drought severity index is formulated in terms of absolute precipitation anomalies, and to allow inter-station comparisons of drought severity, the index is standardised by expressing the accumulated anomalies as a percentage of the median annual precipitation and multiplying the result by -1.

11. Appendix 5: Outline of frequency analysis of drought

The average recurrence interval, ARI, or return period of an observation such as extreme rainfall, drought or flood, is as shown in Appendix 3 the inverse of the annual exceedance probability given by $T(x) = 1/\Pr ob(X \geq x) = 1/(1 - F(x))$, where x is the magnitude of event having an ARI of T, and the random variable X in the above formula is an environmental element such as drought intensity. As droughts do persist for more than one year, traditional extreme value methods of sampling one event per year can not be used. Kim et al. (2003) treat drought duration as independent events from a partial duration series, which can then be converted by standard methods (see for example Madsen et al. (1997)) to an equivalent distribution of annual exceedances. Thus the average recurrence intervals of drought duration becomes

$T_d = 1/\theta[1 - F(d)]$, where $\theta = n/N$ is the total number of droughts n during the observational record of N years. $F(d)$ is the cumulative distribution of drought duration.

For a joint analysis of drought (for example drought duration and drought intensity) the corresponding joint average recurrence interval is given by

$T_{d,i} = 1/\theta[1 - F_{d,i}(d,i)]$, where $F_{d,i}(d,i)$ is the joint cumulative distribution of drought duration and intensity given as $P(D \leq d, I \leq i)$.

An empirical distribution-free probability density function is used to evaluate single and joint distributions of drought. A Gaussian kernel density estimator is used by Kim et al. (2003) to express the probability function which involves weighted moving averages.

For drought duration or other single variable analyses, the empirical density function takes the form

$$\hat{f}_x(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

And for joint distributions where two correlated variables are being analysed, the joint empirical density function is

$$\hat{f}_{xy}(x,y) = \frac{1}{nh_x h_y} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h_x}\right) K\left(\frac{y - y_i}{h_y}\right)$$

where n is the number of observations, h is the bandwidth and is related to the standard deviation of the drought characteristic and K is the Gaussian kernel estimator function.